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Dr. Donald B. Percival

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Applied Physics Laboratory
University of Washington
1013 NE 40th Street
Seattle, WA 98105

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Robert McCoy
Office of Naval Research
Ballston Centre Tower One
800 North Quincy Street
Arlington, VA 22217

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14. ABSTRACT
This report describes work on stochastic models that form the basis for the statistical analysis of clock noise. A popular simple model for clock noise is a pure power law (PPL) process. A method for efficiently computing the autocovariance sequence (ACVS) for discrete parameter stationary PPL processes is developed. The method is extended to handle the ACVS for the stationary process obtained when a nonstationary PPL process is differenced to achieve stationarity.

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Stochastic Models and Statistical Analysis for Clock Noise: Final Report

Donald B. Percival
Applied Physics Laboratory, Box 355640, University of Washington, Seattle WA 98195–5640
phone: (206) 543–1368 fax: (206) 543–6785 email: dbp@apl.washington.edu

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LONG-TERM GOALS

The long-term goal of this project is to enhance the use of high performance clocks by making methodological advances in statistical modeling and analysis.

OBJECTIVES

The objective of this particular study was to advance the use of modern statistical models and analysis techniques on data encountered in the precise time and time interval (PTTI) community. Examples of such models and analysis techniques include (i) fractionally differenced (FD) processes as formulated by Granger and Joyeux [1] and Hosking [2] (these are superior analytically to the continuous parameter power law models in common use today); (ii) multitaper spectral analysis, which offers stable estimates of the power spectrum with low bias [3,4]; and (iii) wavelet variance analysis, which offers important generalizations of the well-known Allan variance for characterizing clock noise [5,6].

APPROACH

Our starting points were the analysis techniques described in two recent papers [7,8], which focus on spectral analysis, wavelet analysis and maximum likelihood estimation as applied to FD and related processes. Our approach was to build upon this work by continuing an investigation into some fundamental properties of discrete parameter power law processes, which provide an analytically tractable alternative to the continuous parameter power law processes commonly used in the PTTI community. We focused on these processes because they form the building blocks for flexible piecewise power law models, which have been used in an *ad hoc* manner in PTTI applications, but have yet to be cast in a rigorous statistical framework. The advantage of discrete parameter models is that they avoid some of the technical pathologies associated with continuous parameter models and are directly useful as models for what is actually observed (i.e., there is no need to adjust the model for a particular sampling rate).

WORK COMPLETED

We presented the results of our work in the form of a tutorial on clock noise [9] and an invited presentation [10] at the 35th Annual Precise Time and Time Interval (PTTI) Meeting, San Diego, California, 1–4 December 2003. We also completed a technical report [11] detailing practical procedures for computing the autocovariance sequence for a

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discrete parameter stationary pure power law process or for a nonstationary process whose increments are such a stationary process. Finally we initiated work on a monograph (joint with C.A. Greenhall, Jet Propulsion Laboratory, and L.S. Schmidt, Rand Corporation) tentatively entitled *The Statistical Analysis of Clock Noise* (the monograph is a long way from completion).

RESULTS

Our main new theoretical result is the derivation of expressions that allow for quick and accurate computation of the autocovariance sequence for a discrete parameter stationary pure power law process. We also derived formulae for the autocovariance sequence of the stationary process that is obtained when a discrete parameter nonstationary pure power law process is appropriately differenced to achieve stationarity. This work makes these processes as attractive computationally as fractionally differenced processes for use as simple models for PTTI applications. This work is also a step in the direction of providing a tractable theory for piecewise power law processes, which, while used in an *ad hoc* manner in the PTTI community, do not have a firm theoretical foundation (i.e., there are no know expressions for the autocovariance sequence for stationary increments of a piecewise power law process, which would be needed for, e.g., optimal forecasting and generating exact simulations).

IMPACT/APPLICATIONS

The results from this project should have a positive impact on the statistical characterization of clock noise, which is a necessary component in evaluating clocks for their use in various systems (including ones associate with the Global Positioning System).

RELATED PROJECTS

None.

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`http://faculty.washington.edu/dbp/PDFFILES/ppl-acvs.pdf`